Analysis of Quantum-Inspired Evolutionary Algorithm

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Abstract- This paper extends the author's previous works on quantum-inspired evolutionary algorithm (QEA). It investigates the characteristics of QEA which is based on the concept and principles of quantum computing such as quantum bit and linear superposition of states. QEA has many advantages such as automatic balance ability between global search and local search, inclusion of individual's past history, having fewer individuals without degrading performance, less computation time, and clearer termination-condition. The experimental results on the knapsack problem are presented to verify these characteristics of QEA.

Keywords: quantum-inspired algorithm, evolutionary algorithm, quantum-inspired evolutionary algorithm, knapsack problem, combinatorial optimization.

1 Introduction

Evolutionary algorithms (EAs) are principally a stochastic search and optimization method based on the principles of natural biological evolution. Compared to traditional optimization methods, such as calculus-based and enumerative strategies, EAs are robust, global and may be generally applied without recourse to domain-specific heuristic. Although EAs lack theoretical background, the experimental results show good performance in many areas. The three mainstream methods of evolutionary computation which have been established over the past thirty years are genetic algorithms (GAs), developed by Holland [1], evolutionary programming (EP), developed by Fogel [2], and evolution strategies (ES), developed by Rechenberg and Schwefel [3].

EAs operate on a population of potential solutions, applying the principle of survival of the fittest to produce successively better approximations to a solution. At each generation of the EA, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and reproducing them using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals from which they were created, just as in natural adaptation.

However, EAs have several disadvantages such as poor balance ability of exploration and exploitation, and absence of individual's past history. In other words, a strong selective pressure supports the premature convergence of the search; a weak selective pressure can make the search ineffective [4]. And there is no past history of the information of evolving individuals in EAs. In [5], we have already proposed the basic structure of a quantum-inspired evolutionary algorithm (QEA) which could solve these problems, and also proposed a parallel quantum-inspired evolutionary algorithm for combinatorial optimization problems in [6]. The experimental results on the knapsack problem demonstrated the effectiveness and the applicability of QEA [5, 6].

This paper investigates the characteristics of QEA. How does QEA work?; Why does QEA have good performance?

This paper is organized as follows. In Section 2, we review the previous work of quantum-inspired evolutionary algorithm (QEA). Section 3 contains a description of the kanpsack problem. In Section 4, we investigate the characteristics of QEA. Concluding remarks follow in Section 5.

2 Quantum-inspired Evolutionary Algorithm (QEA)

QEA is based on the concepts of quantum bits and superposition of states [5]. The smallest unit of information stored in a two-state quantum computer is called a quantum bit or qubit [7, 8, 9]. A qubit may be in the '1' state, in the '0' state, or in any superposition of the two. The classical representation can be broadly classified as: binary, numeric, and symbolic [10]. QEA uses a new representation that is based on the concept of qubits. QEA with the qubit representation has a better characteristic of diversity than classical approaches, since it can represent superposition of states. Convergence can also be obtained with the qubit representation. As a qubit approaches to 1 or 0, the qubit chromosome converges to a single state and the property of diversity disappears gradually. That is, the qubit representation is able to possess the two characteristics of exploration and exploitation, simultaneously.

The basic structure of QEA is described in the following.

procedure QEA
begin
$t \leftarrow 0$
initialize $Q(t)$
make $P(t)$ by observing $Q(t)$ states
evaluate $P(t)$
store the best solution among $P(t)$

while (**not** termination-condition) **do**
begin
$t \leftarrow t+1$
make $P(t)$ by observing $Q(t-1)$ states
evaluate $P(t)$
update $Q(t)$ using quantum gates $U(t)$
store the best solution among $P(t)$
end
end

where $Q(t)$ is a population of qubit chromosomes at generation t, and $P(t)$ is a set of binary solutions at generation t.

In the step of 'initialize $Q(t)$,' all qubit chromosomes are initialized with the same constant. It means that one qubit chromosome represents the linear superposition of all possible states with the same probability. The next step makes a set of binary solutions, $P(t)$, by observing $Q(t)$ states. One binary solution is formed by selecting each bit using the probability of qubit. And then each solution is evaluated to give some measure of its fitness. The initial best solution is then selected and stored among the binary solutions, $P(t)$.

In the **while** loop, one more step, 'update $Q(t)$,' is included to have fitter states of the qubit chromosomes. A set of binary solutions, $P(t)$, is formed by observing $Q(t-1)$ states as with the procedure described before, and each binary solution is evaluated to give the fitness value. In the next step, 'update $Q(t)$,' a set of qubit chromosomes $Q(t)$ is updated by applying some appropriate quantum gates $U(t)$, which is formed by using the binary solutions $P(t)$ and the stored best solution. The appropriate quantum gates can be designed in compliance with practical problems. Rotation gate is used as a basic gate of QEA. This step makes the qubit chromosomes converge to the fitter states. The best solution among $P(t)$ is selected in the next step, and if the solution is fitter than the stored best solution, the stored solution is replaced by the new one. The binary solutions $P(t)$ are discarded at the end of the loop.

3 Knapsack Problem

The knapsack problem, a kind of combinatorial optimization problem, is used to investigate the characteristics of QEA. The knapsack problem can be described as selecting from among various items those items which are most profitable, given that the knapsack has limited capacity. The 0-1 knapsack problem is described as: given a set of m items and a knapsack, select a subset of the items so as to maximize the profit $f(\mathbf{x})$:

$$f(\mathbf{x}) = \sum_{i=1}^{m} p_i x_i,$$

subject to

$$\sum_{i=1}^{m} w_i x_i \leq C,$$

where $\mathbf{x} = (x_1 \cdots x_m)$, x_i is 0 or 1, p_i is the profit of item i, w_i is the weight of item i, and C is the capacity of the knapsack.

4 Characteristics of QEA

The knapsack problem described before was used to analyze the characteristics of QEA. For the purpose of the analysis, the knapsack problem with 10 items was considered in this Section. While selecting a subset from the 10 items, there exist 2^{10} cases. Figure 1 shows the profit value of 1024 cases in the knapsack problem. For this problem, the best profit is 62.192938 at the 127th. So as to investigate the characteristics of QEA, we set the population size of the qubit chromosome at one. Figure 3 shows the probability of 1024 cases using the qubit chromosome at generation 10, 20, 30, 40, 50, 100, 200, and 300. In (a) and (b) of Figure 3, the horizontal lines valued at about 0.001 show the initialized probability of all the solutions. It means that the initialized qubit chromosome includes all solutions with the same probability. That is, it means that QEA starts with a random search initially.

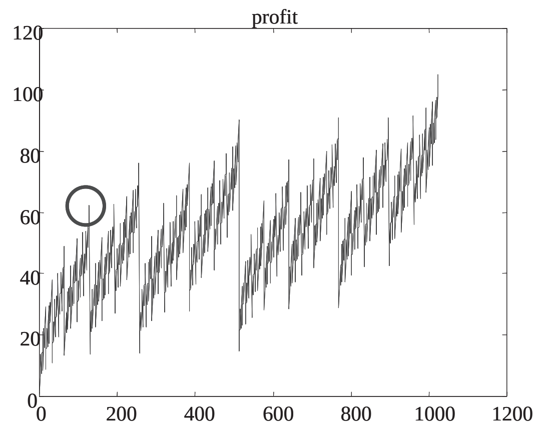

Figure 1: Profit value of 1024 cases in the knapsack problem with 10 items. (The best profit is marked with O.)

The result at generation 10 was interesting. Even though the size of qubit chromosome was one, the probability of 1024 cases had a similar pattern to the profit value of Figure 1. That is, the only one chromosome was able to include the information of 1024 cases. At generation 20, the probability increased on the whole. At generation 30 to 50, the probability of the cases with large profit increased on a large scale. At generation 100, however, all peak value decreased except the peak of the best solution. The same features were obtained at generation 200. At generation 300, the probability of the best solution was over 0.9, and that of the other solutions was about 0. That is, the qubit chromosome had almost converged to the best solution.

The results above can be summarized as follows. Initially, QEA starts with a random search, and the pattern of all cases is then caught gradually. The probability of the cases with large profit increases, and it changes into a local search step by step. Finally, the probability of the qubit chromosome converges to the best solution. That is, QEA starts with a

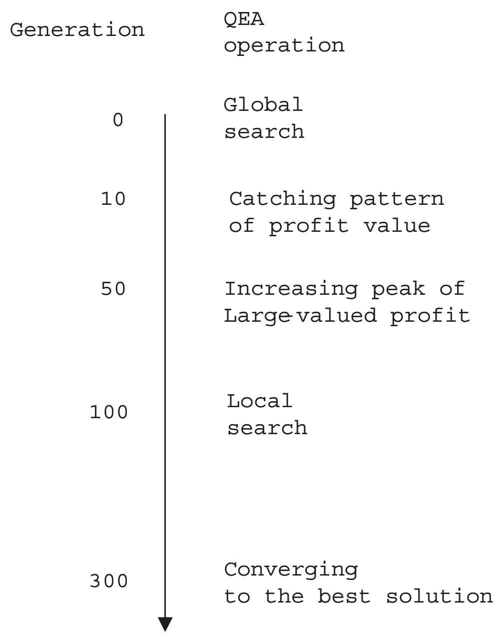

Figure 2: QEA operation.

global search and changes automatically into a local search because of its structural characteristics. Since a qubit chromosome can include the individual's past history, the pattern of all cases can be caught in spite of starting with one chromosome.

The characteristics of QEA can be summarized as follows.

(a) Automatic balance ability between global search and local search (due to the characteristics of QEA structure).

(b) Inclusion of individual's past history (due to the probabilistic representation).

(c) Fewer individuals without loss of performance (due to the linear superposition of the states).

(d) Less computation time.

(e) Clearer termination-condition.

(for example, $Prob(b) > \gamma$, where $0 < \gamma < 1$ and $Prob(b)$ can be calculated easily by using the probability of a qubit chromosome.)

5 Conclusions

This paper investigated the characteristics of QEA which is based on the concept of quantum computing such as qubit and superposition of states. QEA can represent linear superposition of states, and there is no need to include many individuals. QEA has excellent global search ability due to its diversity caused by the probabilistic representation, and possesses fast convergence to the best solution due to the inclusion of individual's past history. It also has a clearer termination-condition and less computation time. The experimental results on the knapsack problem verified these characteristics of QEA.

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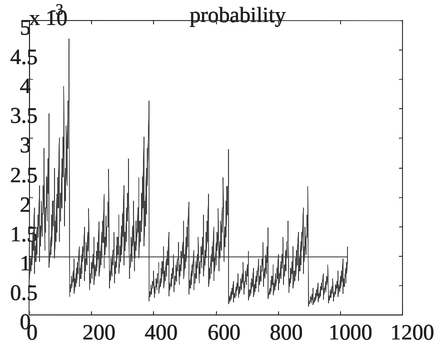
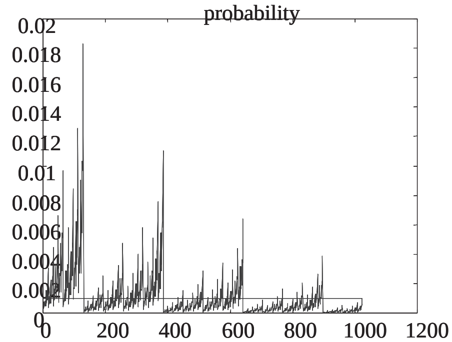
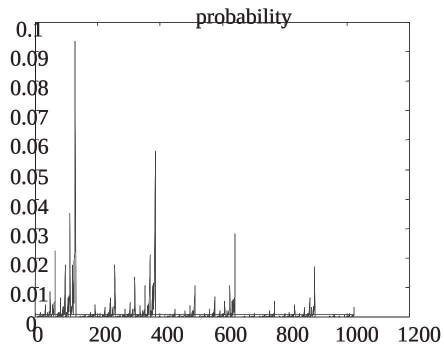


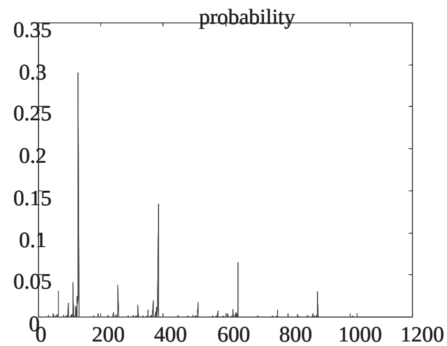
(a) Generation 10



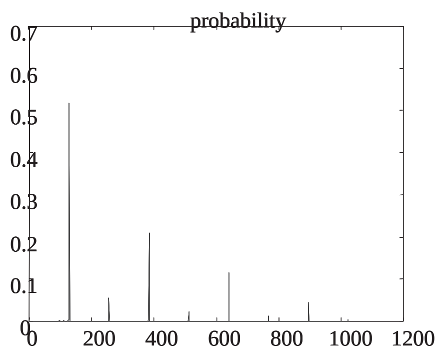
(b) Generation 20



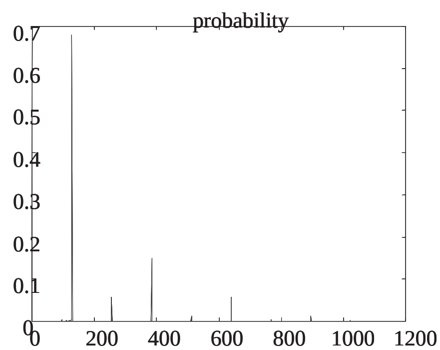
(c) Generation 30



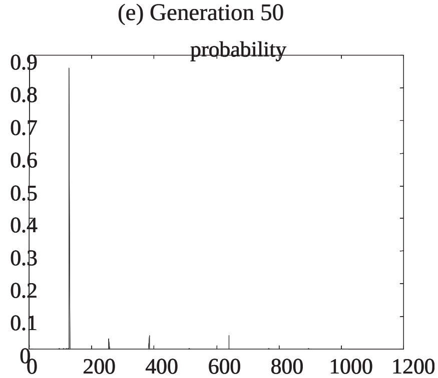
(d) Generation 40



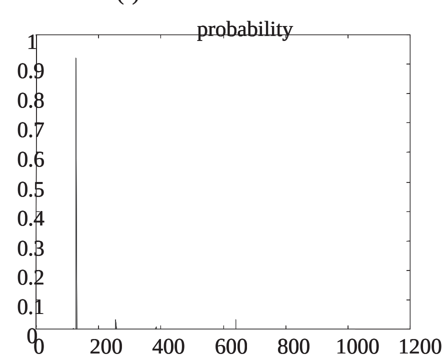
(e) Generation 50



(f) Generation 100



(g) Generation 200



(h) Generation 300

Figure 3: Probability of all solutions using qubit chromosome.